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An ANALOG COMPUTER for STUDYING HEAT TRANSFER during a THERMAL RECOVERY PROCESS

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ABSTRACT

A design is presented for an electrical analog computer which can solve non-steady state heat transfer problems in an extensive radial formation containing a moving cylindrical source. The computer is used to simulate a simplified thermal oil recovery process in which heat transfer from a moving, constant temperature source takes place radially by conduction only. Temperature distribution curves are shown for several different assumed modes of travel for the heat source. The data are used to estimate the residual fuel requirements necessary to maintain a self-propagating isothermal front for the particular system being studied.

Although the computer is designed to represent an unique system, conversion factors can be adjusted to show the effects of changing the assumed values of system constants, such as thermal conductivity of the formation and temperature of the source isotherm. Examples are given to illustrate the relative influence the assumed values of these variables will have upon the quantity of fuel required to maintain the source.

The maximum effect of heat transfer by the flowing air stream is estimated mathematically, and shown to be markedly reduced.

Inasmuch as the examples discussed refer to a simplified process, the results are not directly applicable to a practical field operation. It is believed, however, that the data illustrate some general trends which are important in thermal recovery processes.

Data from laboratory and field experimentation can be used to modify the computer to take into account the influence of heat transfer by various mechanisms, for example, by injected air and by fluids ahead of the hot zone. With such refinements it would be possible to estimate more accurately the limiting values for the rate of travel of the high temperature front and for the required air injection rates.

Use of this computer should aid in evaluating the economic feasibility of oil recovery by various thermal processes. It is hoped this paper will stimulate further work by others to help accomplish this objective.

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INTRODUCTION

As the costs of exploration for and development of new oil fields rise, increasing the recovery of oil from established fields becomes more and more important. In fields containing low gravity crude oil, ultimate recoveries by primary means are sometimes less than 10 per cent and often total only 10 to 20 per cent of the oil in place. Conventional secondary recovery methods are not always effective in increasing this total. Methods which have been proposed for increasing the recovery of viscous oils include thermal processes for increasing the mobility of the oil. One method is to provide a moving heat source generated within the formation to heat the oil and a flowing gaseous medium (for example, air) to sweep the oil to the producing wells. The possibility of such a process has intrigued engineers for many years.[1,2,3] but little or no information has been available in the literature relative to the technical or economic feasibility of this method. Recently, however, interest has been revived by publication of laboratory and field test results indicating the feasibility of maintaining

[1]References given at end of paper.

a self propagating heat source in an oil formation. Very high ultimate recoveries were indicated to be possible[4,5,6].

To date, numerous laboratory tests have been devoted to studying the many variables involved, such as oil and oil-sand characteristics, pressure, and air flux. Of necessity, such tests are usually made in linear cells. The application of laboratory results to a radial system of sufficient scale to evaluate the economic feasibility of a thermal oil recovery process is extremely expensive and subject to considerable trial and error. One intermediate step is to use mathematical and analog methods to study means of utilizing the experimental information in the most effective manner.

To a large extent, the evaluation of economic feasibility depends upon a knowledge of such factors as sweep-out efficiency, well spacing, and air injection rates and pressures. A study of areal sweep of a combustion pattern has been reported in a recent paper by H. J. Ramey, Jr., and G. W. Nabor.[7]

It is proposed in the present paper to describe a design for an electrical analog computer which can calculate the temperature profile resulting from non-steady state heat transfer from a moving source in an extensive radial system. A method is presented for using such profiles to estimate the fuel required to maintain a moving heat source. The effect of changes in the assumed values of different variables is demonstrated through changes in scale factors in the computer.

In the present study the computer was used to simulate a simplified thermal process in which the single factor of heat transfer by conduction alone was considered. Inasmuch as the nature of the thermal oil recovery process may predetermine the quantity of fuel available for generating the heat source, and the heat transfer characteristics are dependent upon the physical operating conditions, it may be concluded that the optimum mode of travel of the source will be affected by additional factors not included in the present design. Through the utilization of data obtained in laboratory and field experiments, the computer described herein may be modified to evaluate different hypothetical mechanisms for the recovery of oil by thermal means.

APPARATUS AND PROCEDURE

Mathematical or experimental solutions of unsteady state flow problems are often difficult and time consuming. A common method of obtaining a rapid solution to such problems is often found by use of electrical analogs.[8,9,10] One such network[11] has been adapted in our laboratory to study heat conduction from a moving, cylindrical heat source in the earth.

The electrical model makes use of the similarities which exist between the flow of heat in a rigid body and that of charge in a non-inductive electrical circuit.

The method is based on the identity in form between the following electrical and heat flow equations:

Heat Flow	Electrical Flow
$R_t = \dfrac{\Delta T}{q}$	$R_e = \dfrac{\Delta V}{I}$
$C_t = \dfrac{Q_t}{\Delta T_{AVG}}$	$C_e = \dfrac{Q_e}{\Delta V_{AVG}}$

It follows from this identity in form of the fundamental defining equations that all mathematical results based on them will be similar.

In the analog computer discussed in this paper, the simulated formation was divided into 100 concentric, cylindrical tubes of unit thickness, each varying from the next one by an increment of radius. Because of the symmetry of this two-dimensional problem, it was possible to simplify the electrical network by adjusting the values of the components to represent the thermal capacitance and resistance associated with the radial location of each thermal element. To approximate an infinitely large medium, the last element in the circuit was designed to correspond to a formation volume which was large compared to the entire radial system being studied.

In order that the computer could be operated at a convenient voltage and transient time interval, scale factors were chosen for the conversion from thermal to electrical units, as follows:

$l = 0.18$ v/°F; that is, 180 v is equivalent to 1000°F.

$n = 0.006$ sec/hour; that is, one minute in the computer is equivalent to 10,000 hours actual time.

$m = 3.04 \times 10^{-10}$ farads/Btu/°F; that is, one microfarad is equivalent to 3290 Btu/°F.

The following thermal constants were assumed to be representative of a fluid saturated, porous rock:

thermal conductivity, $k = 1.6$ Btu/sq ft hour °F/ft
specific heat, $c = 0.2$ Btu/lb °F
density, $\rho = 140$ lb/cu ft
diffusivity, $\alpha = 0.057$ sq ft/hour

The scale factors may easily be changed without invalidating the results as long as the relationships between the two systems (electrical and thermal) are maintained. It is possible, within certain practical limits, to study the effect of changing the assumed values of the variables in the thermal systems by making suitable changes in the scale factors. This is important in considering the usefulness of the computer.

For a given position (x) in the thermal system, the thermal resistance is:

$$\Delta R_t = \frac{1}{k\,A_x/\Delta r_x}$$

$$= 0.625\,\frac{\Delta r_x}{A_x}\,\frac{\text{°F hour}}{\text{Btu}}$$

To convert to electrical units:

$$\Delta R_e = \frac{n}{m}\Delta R_t =$$

$$1.233 \times 10^7\,\frac{\Delta r_x}{A_x}\text{ ohms}$$

where A_x is the logarithmic mean area.

For the same element, the thermal capacitance is:

$$\Delta C_t = \pi(r_{x+1}^2 - r_x^2)\cdot 1\cdot\rho\cdot c$$

$$= 87.6\,(r_{x+1}^2 - r_x^2)\text{ Btu/°F}$$

To convert to electrical units:

$$\Delta C_e = m\Delta C_t =$$

$$2.66 \times 10^{-8}\,(r_{x+1}^2 - r_x^2)\text{ faradays}$$

A schematic diagram of the computer circuit is shown in Fig. 1. The body of the computer, a conventional RC network made of non-inductive resistors and oil-filled condensers, is shown with each of the elements connected to the bars of a large commutator. The voltage source is a 180 v "B" battery, chosen to give full scale deflection of the oscilloscope beam when connected directly to the deflection plates.

To simulate the movement of a heat source, the voltage source is connected to a rotating brush which contacts the commutator bars. A second brush, rotating at a high speed relative to the primary voltage source, is used to transfer the condenser voltage at each element to the oscilloscope plates. By means of an adjustable selector switch synchronized with the source rotor, a triggering signal is sent to a commutator bar adjacent to the last RC element to initiate the oscilloscope sweep. This results in a visual representation of the voltage profile in the computer network for the selected location of the simulated heat source. A photograph of the assembled computer and associated equipment is shown in Fig. 2.

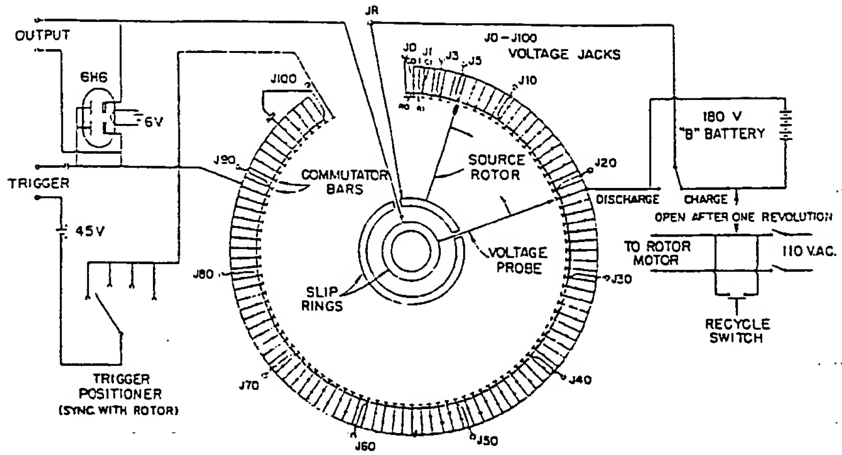

FIG. 1 — SCHEMATIC DIAGRAM OF ANALOG COMPUTER FOR HEAT TRANSFER BY CONDUCTION IN A RADIAL FORMATION.

To operate the computer, the rate of travel of the source rotor is selected by changing the gear ratio on the drive motor. After a suitable warm-up period, the positions of the trigger selector switch are adjusted and synchronization is checked with the oscilloscope sweep circuits. The camera is attached to the oscilloscope and the network voltage profile photographed at the preselected positions.

If the computer is used to simulate a constant temperature source moving through a radial formation, a temperature profile similar to that shown in Fig. 3 is observed on the oscilloscope screen. The vertical distance between grid lines represents a temperature difference of 100°F. The curvature of the grid lines calibrates the beam deflection for the entire screen. The voltage of each segment of the computer network is shown as a spot on the screen at a vertical position corresponding to its numerical value.

This analog computer can be adapted to the solution of heat transfer problems involving steady state or non-steady state conditions and moving or stationary heat sources. In order to check the network design, analog solutions were determined for problems involving a stationary heat source for which the theoretical curves could be obtained by conventional mathematical methods. In general, it was concluded that the network satisfactorily represented a cylindrical system and that the solution of non-steady state problems in a simulated infinite cylindrical medium was feasible for short times. For problems involving long computing times at a given position of the source, the error increases with time.[12] For such problems the accuracy could be improved by using a DC amplifier at each lump in the analog.[13,14,15] However, for problems involving a moving source, the large additional expense involved in such a refinement was not considered necessary.

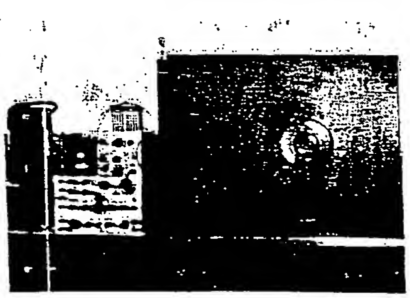

FIG. 2—ASSEMBLED COMPUTER AND ASSOCIATED EQUIPMENT.

As described herein, the computer has been designed to study heat conduction problems involving a moving, constant temperature source in a radial formation. It should be pointed out, however, that the general design is applicable to a variety of heat transfer problems involving a moving source. With some modification it would be possible to study non-steady state heat transfer by gases, liquids, and solids, either individually or as an interacting system, in media of different geometrical shapes. In addition, the moving source rotor may be designed to represent a heat source with any of a number of unique properties.

ILLUSTRATIVE PROBLEMS

To illustrate the type of problem which can be studied with this computer, several assumed modes of travel of the heat source will be discussed. It should be emphasized that these problems are intended to be illustrative only and are not necessarily representative of a practical field operation.

For the following illustrations, it is assumed that a constant temperature heat source is produced in the formation. Although a number of possible mechanisms for the production of such a heat source may be visualized, for convenience a mechanism similar to that postulated by Kuhn and Koch[4] has been assumed. According to their hypothesis, a combustion zone is produced by burning residual carbonaceous material deposited out of the native crude oil ahead of the high temperature zone. In the idealized case, the temperature of this combustion zone could be maintained at a constant optimum value by adjustment of the rate of air injection into the formation. Although reference will be made throughout the ensuing discussion to the "combustion front" and the associated fuel requirements, any means of obtaining a constant temperature front would result in identical heat transfer characteristics for the particular conditions postulated.

In one set of tests, a constant temperature source was moved radially through the formation at various constant speeds which ranged from 0.874 ft/day to 0.097 ft/day. Temperature profiles were photographed at convenient positions. In Figs. 4, 5, 6, and 7, temperature data from the above tests have been plotted on a semi-log scale to show the computed temperature distributions for different positions of the heat source. It is immediately apparent that (1) at a constant rate of travel, preheating of the formation ahead of the source isotherm increases with distance of travel and (2) at a given radius, preheating increases for slower rates of travel.

These temperature distribution curves may be used to calculate the theoretically required amount of residual fuel to maintain a self-propagat-

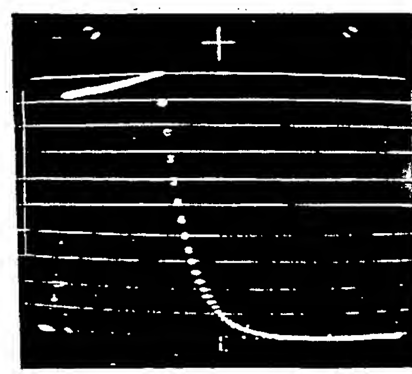

FIG. 3—TYPICAL VOLTAGE PROFILE IN ANALOG COMPUTER NETWORK.

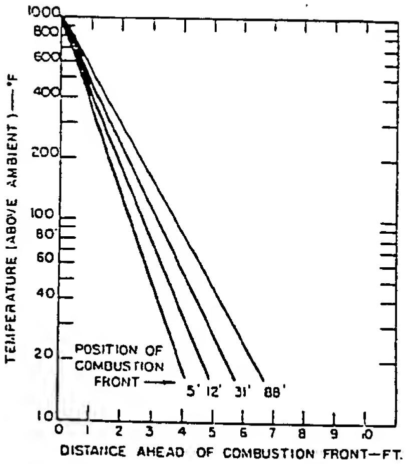

FIG. 4 — TEMPERATURE DISTRIBUTION AHEAD OF COMBUSTION FRONT. **Conduction only in radial formation. Rate of movement of front: 0.874 ft/day.**

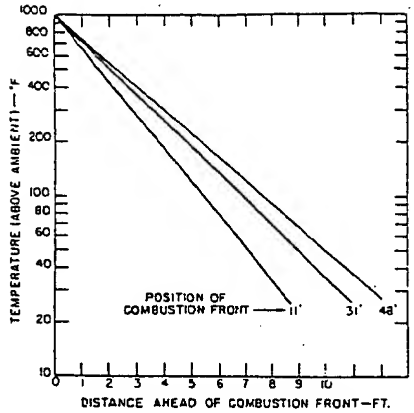

FIG. 5 — TEMPERATURE DISTRIBUTION AHEAD OF COMBUSTION FRONT. **Conduction only in radial formation. Rate of movement of front: 0.353 ft/day.**

ing combustion front at a given rate of advance. In order to make such a calculation, it is necessary to know the instantaneous heat loss from the combustion front. If this heat loss can be supplied by burning the available fuel, the temperature of the combustion front will remain constant and the source will be self-perpetuating as long as sufficient air is supplied. If there is insufficient fuel, the temperature will decrease and, under certain conditions, the process will cease.

The instantaneous heat loss can be determined from the equation, $q = kA\ \partial T/\partial r$. The term, $\partial T/\partial r$, is simply the instantaneous temperature gradient at the heat source and can be readily determined directly from the figures as

$$\partial T/\partial r = \frac{\ln T_1 - \ln T_2}{\Delta r \Big]_{T_1}^{T_2}} \cdot T$$

Then the fuel required to maintain the combustion front will be

$$W = \frac{k}{s} \cdot \frac{\partial T/\partial r}{dr/dt} \text{ lb/cu ft of formation}$$

where s is the heat of combustion of the residual fuel (assumed as 17,000 Btu/lb) and dr/dt is the rate of movement of the front in feet per hour.

Through use of the above method, Fig. 8 has been plotted to show the residual fuel required to maintain a moving heat source (temperature = 1,000°F above ambient) in a formation in which heat is transferred radially by conduction alone.

A suitable change in conversion factors (l = 0.18, n = 0.006, m = 6.08 × 10^{-10}, $r = r_x/\sqrt{2}$), can be made to show the effect of changing the assumed value of the conductivity. In Fig. 9 the fuel requirements to maintain a moving combustion front are indicated for k = 0.8, ΔT = 1,000.

Similarly, through the choice of a different set of conversion factors (l = 0.30, n = 0.006, m = 6.08 × 10^{-10}, $r = r_x/\sqrt{2}$), the effect of a change in the temperature of the combustion front can be found. Fig. 10 shows the fuel requirements if k = 0.8, ΔT = 600.

In general, Figs. 8, 9, and 10 show that the fuel requirements are higher for the slower rates of travel of the source. Inasmuch as the physical nature of the assumed process makes available only a given amount of fuel, which may be affected by formation and oil characteristics, there should be a minimum rate of advance possible for a self-propagating combustion front. The curves indicate that this critical rate will decrease with distance from the wellbore. This characteristic is a result of the greater preheating by conduction as the combustion front travels away from the wellbore and should be true when the additional effect of injected air is considered.

From the data presented above, it is apparent that the differences between the actual value of k and that used in the computer are not critical in determining the required fuel content, provided k is of the right order of magnitude. Of greater importance in using the data is the value assumed for the temperature at the combustion front, inasmuch as the required fuel content was shown to be directly proportional to the temperature. This indicates one area in which experimental data from laboratory tests would be desirable.

In order to maintain a constant rate of advance of the combustion front as shown in these figures, it would be necessary to provide for an increasing volume rate of injection air as the front travels away from the wellbore. To determine the required air rates, it would be necessary to know the position of the front at any time. This is not likely to be practicable for field operation. A simpler operational method would be to inject air at a constant rate. If both the fuel supply per unit volume of formation and the air injection rate are assumed to be constant, it would appear that because of geometrical factors the minimum rate of movement of the combustion front should be inversely proportional to the radial position.

It is possible to study the feasibility of this mode of operation by either of two methods. In one case, a family of curves, each of which is proportional to 1/radius, can be obtained with a constant rate drive. In the second case, the computer can be set up so that the source rotor can be moved at a rate proportional to 1/radius.

To illustrate the first method, the family of curves in Fig. 8 has been used to examine a mode of travel in which the rate of movement of the combustion front is equal to 8/radius in feet per day. Then for the constant rates of 0.874, 0.353, 0.211 and 0.097 ft/day, the front would be located at approximately 9, 23, 38 and 83 ft, respectively, in order to be traveling at the designated rate. If these points are plotted on Fig. 8 they will all fall at a constant fuel requirement of approximately 2.3 lb/cu ft of formation, indicating that the front would be self-propagating

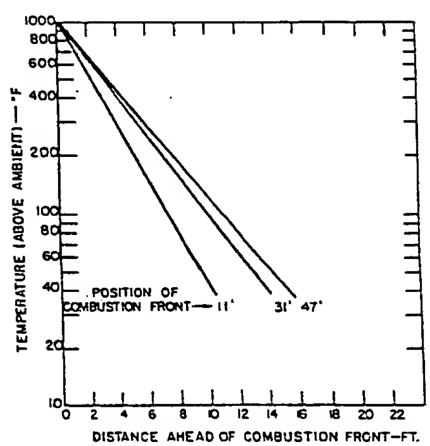

FIG. 6 — TEMPERATURE DISTRIBUTION AHEAD OF COMBUSTION FRONT. **Conduction only in radial formation. Rate of movement of front: 0.211 ft/day.**

under the above conditions.

If the computer is set up so that temperature profiles are obtained at radii, r_i, at the corresponding rate, $8/r_i$, the required residual fuel can be calculated from the computed temperature gradients in the manner described previously. In this case a single curve would be obtained as shown in Fig. 11. Here again the required residual fuel is found to be 2.3 lb/cu ft of formation.

In general, it may be concluded that for this mode of travel the numerical value of both the temperature gradient and the radial velocity of the combustion front must be changing at the same rate. The results indicate also that in the radial system described a constant rate of fuel consumption will result in a constant temperature heat source only if the rate of movement is inversely proportional to the radial position.

In the above discussion, heat transfer by the injected air has been neglected. It is possible, however, to use the available data to estimate the minimum fuel requirements in a system in which no heat is retained behind the combustion zone. This can be done by assuming that all of the heat stored in the rock as the moving source reaches and passes a given radius will be transferred instantaneously by the flowing air stream back to the combustion front. Thus the minimum fuel requirement in such a system can be approximated simply by subtracting the equivalent fuel content of the corresponding volume increment of heated rock from the residual fuel content calculated from the data shown in Figs. 4, 5, 6, and 7. This has been done for a radial thermal system in which $k = 1.6$ and ΔT = 1,000 and the calculated

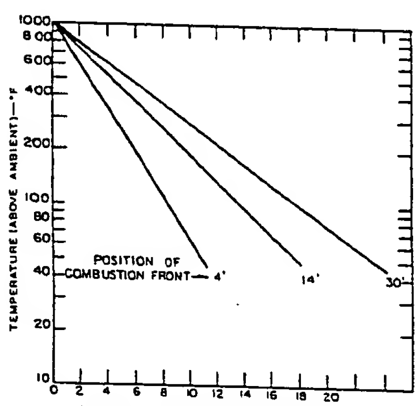

FIG. 7 — TEMPERATURE DISTRIBUTION AHEAD OF COMBUSTION FRONT. Conduction only in radial formation. Rate of movement of front: 0.097 ft/day.

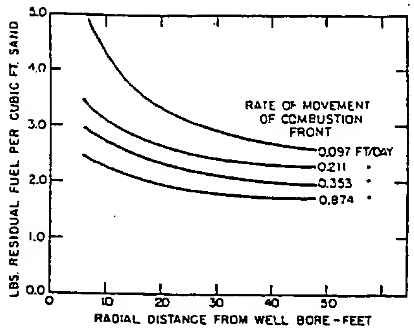

FIG. 8 — FUEL REQUIRED TO MAINTAIN A MOVING COMBUSTION FRONT. Thermal conductivity: 1.6 Btu/ft² hour °F/ft. Combustion front 1,000°F above Ambient Temperature.

curves have been plotted in Fig. 12. The values for the residual fuel content shown in Figs. 8 and 12 represent two extremes of operation. In the one case (Fig. 8), it is assumed that all of the heat stored in the rock behind the combustion front remains unavailable for raising the temperature of the formation. In the second case (Fig. 12), all of this heat is utilized to the fullest extent.

As a final illustration of the usefulness of the analog, there is shown in Fig. 13 the analog computation of the temperature decay in a radial formation after a heat source has stopped moving. In this problem, the combustion front was moved through the formation at a constant rate of 0.353 ft/day to a radius of 40 ft and then stopped. This might correspond to a system (with heat flow by conduction only) in which air injection has stopped because of equipment failure. From the slow decay of temperature shown, it might be concluded that once a hot zone has been well established, re-ignition of the oil sand following a prolonged shutdown should not be a serious problem.

This problem is similar in nature also to that investigated mathematically by Jenkins and Aronofsky.[10] The temperature increase after 840 days of 70° F at a radius of 100 ft compares reasonably well with their theoretical value of 75° F after two years.

From the above illustrations, it would appear that the computed temperature distribution curves should be useful for studying various factors, such as the best start-up technique and the optimum mode of travel of the heat front. It is obvious, however, that the simplified mechanism set up for the foregoing problems does not simulate closely the heat transfer process existing in a practical field operation. In order to obtain the maximum utility from this machine,

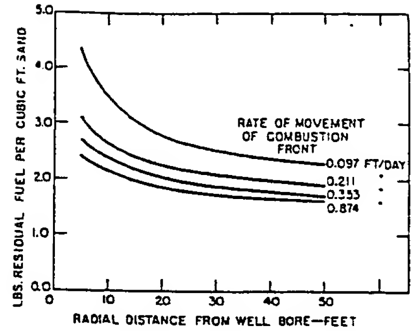

FIG. 9 — FUEL REQUIRED TO MAINTAIN A MOVING COMBUSTION FRONT. Thermal conductivity: 0.8 Btu/ft² hour °F/ft. Combustion front 1,000°F above Ambient Temperature.

it would be desirable to modify it to include additional factors such as heat transfer by the injected air and by the movement of hot fluids ahead of the combustion front. It seems apparent that modification of the computer to study heat transfer by various assumed mechanisms must be coordinated closely with results obtained from laboratory and field experimentation.

CONCLUSIONS

A design has been presented for an electrical analog computer which can solve non-steady state heat conduction problems pertaining to an extensive radial system containing a moving, cylindrical heat source.

It has been the intent of this paper to show the type of problem the analog computer can be adapted to study, and to indicate how its use can be integrated with the results obtained from laboratory data to increase our understanding of possible field applications of oil recovery by thermal methods. A method has been shown for using the temperature distribution curves obtained with the computer to calculate the fuel required to maintain a self-propagating heat source.

Inasmuch as the computer design presented herein takes into account only those heat losses resulting from radial conduction, the data should not be construed as applying directly to field operations. Modification of the computer to include other heat transfer mechanisms depends primarily upon an understanding of the particular thermal oil recovery method being studied. Laboratory scale studies of the thermal process itself aid in determining the most satisfactory mechanism to use.

Because of the complex nature of thermal oil recovery processes, it

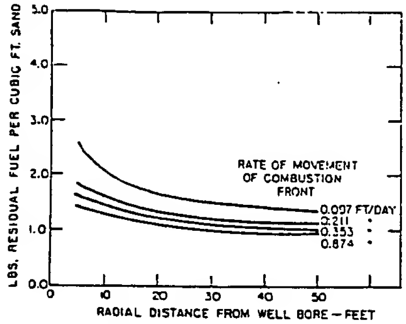

FIG. 10—FUEL REQUIRED TO MAINTAIN A MOVING COMBUSTION FRONT. **Thermal conductivity: 0.8 Btu/ft² hour °F/ft. Combustion front 600°F above Ambient Temperature.**

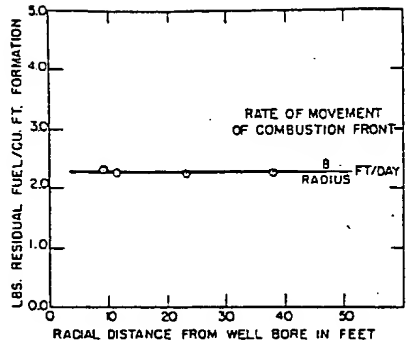

FIG. 11—FUEL REQUIRED TO MAINTAIN A MOVING COMBUSTION FRONT. **Thermal conductivity: 1.6 Btu/ft² hour °F/ft. Combustion temperature 1,000°F above Ambient.**

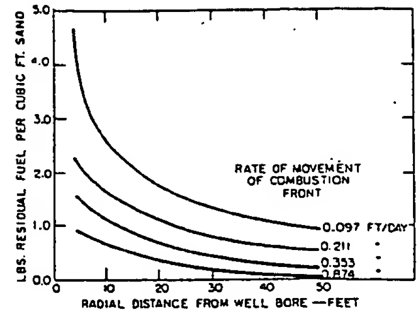

FIG. 12—MINIMUM FUEL REQUIRED TO MAINTAIN A MOVING COMBUSTION FRONT. **Heat transfer by conduction and no heat stored behind the front. Thermal conductivity: 1.6 Btu/ft² hour °F/ft combustion front 1,000 °F above Ambient temperature.**

would appear that continued efforts of research workers in many different groups will be required to evaluate the numerous possible ramifications in a reasonable time. It is hoped that this paper will stimulate the publication of other work on thermal recovery processes.

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NOMENCLATURE

Thermal units

R_t = thermal resistance, °F hour/Btu
C_t = thermal capacity, Btu/°F
Q_t = heat stored in C_t, Btu
q = hourly heat flow, Btu/hour
ΔT = rise in temperature of C_t due to Q_t, °F
r_t = distance in the thermal circuit (radial), ft
A_n = average cross sectional area of each increment Δr_n, sq ft
N = number of the element (Δr) under consideration
t_t = time in the thermal circuit, hour
$\alpha = \frac{k}{c\rho}$ = thermal diffusivity, sq ft/hour
c = heat capacity per unit mass, Btu/lb °F
k = thermal conductivity, Btu/sq ft hour °F/ft
ρ = density of formation, lb/cu ft

Electrical units

R_e = electrical resistance, ohms
C_e = electrical capacity, farads
Q_e = charge stored in C_e, coulombs
I = electrical current, amperes
ΔV = rise in voltage of C_e due to Q_e, volts
t_e = time in the electrical circuit, seconds
r_e = distance in the electrical circuit (radial), ft

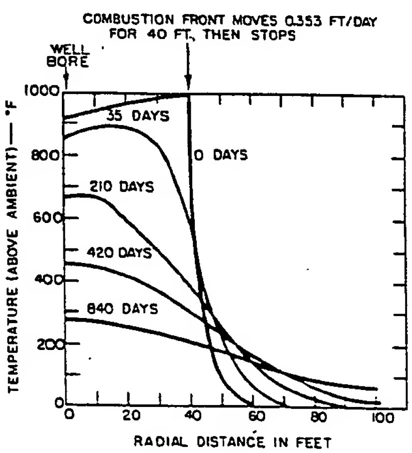

FIG. 13 — CONDUCTION OF HEAT AFTER A COMBUSTION FRONT HAS BEEN STOPPED.

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DISCUSSION

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The authors are to be complimented for their ingenious solution to a problem having an important bearing on future development of thermal oil recovery processes. The interpretation of results from this heat transfer study relative to an in-situ combustion oil recovery process illustrates the utility of this type of theoretical work. Undoubtedly, their electric analog computer will have many other uses in reservoir technology.

With regard to the results of the study, it should be emphasized that for the more conventional well patterns, radial flow conditions exist for only 20 to 30 per cent of the distance from an injection well toward the nearest production well. The results from this study are, therefore, most applicable and important in consideration of the ignition phase of this type of thermal oil recovery process. As the authors implied, the fuel requirements presented are based on assumed thermal properties and an idealized mechanism. Actual fuel consumption can differ greatly from the theoretical values obtained from this study. Nevertheless, the trends illustrated by the authors permit important observations.

For the authors' case of a constant combustion front velocity, the minimum fuel requirement appears especially critical when the burning front is near the injection well and/or when the front is moving at a slow rate. In contrast, the results for the case where the velocity varies inversely with radius of the burning front indicate a fuel requirement that does not vary with position of the burning front. While this result tends to contradict the conclusions drawn from the case of constant velocity, it is believed that most actual operations will fall between these conditions, and the fuel requirement will be substantially higher near the injection well.

During investigation of this thermal recovery process, we computed fuel requirements by an analytical solution of substantially the same heat conduction problem. Two cases were considered: (1) heat conduction from an infinite cylindrical source moving radially in an infinite medium of constant thermal properties, and (2) the same problem with the inclusion of the boundary conditions resulting from consideration of the ignition wellbore.

The solutions to Case (1) for a burning front velocity inversely proportional to the radial position of the front showed the fuel requirement to be entirely independent of position of the front. And for the same thermal conditions assumed by Vogel and Krueger, computed results were in good agreement with the values from the electrical analogy. A fuel requirement of 2.4 lbs/cu ft was obtained as compared with 2.3 reported by Vogel and Krueger. This establishes a direct verification of the electrical analog for a moving source.

A solution for Case (2) at a radial position of 0.5 ft indicated a slightly higher fuel requirement of 2.7 lbs/cu ft. But this is attributed to the effect of the wellbore.

The analytical solutions mentioned above are as follows.

1. Case (1) — Constant velocity of source ($r' = Ut'$).

$$T - T_o = \frac{Uq}{2k}\int_0^t \frac{t'}{(t-t')} e^{-\frac{(r^2+U^2t'^2)}{4\alpha(t-t')}} I_o\left\{\frac{rUt'}{2\alpha(t-t')}\right\} dt' \quad . \quad . \quad . \quad . \quad (1)$$

Where T_o is the initial temperature, T is the temperature at radius r at time t, q is the rate of heat generation per square foot of frontal area, U is the constant velocity of the burning front, k is the thermal conductivity, t is time, and t' is the time the burning front is located at radius r', α is the thermal diffusivity, and I_o is Bessel's function of the first kind of an imaginary argument. If Equation 1 is integrated for $r = r'$ at t', the fuel requirements may be found from the expression:

$$\frac{\text{lb fuel}}{\text{cubic foot}} = \frac{q}{Us}, \quad . \quad . \quad . \quad . \quad (2)$$

where s is the heat of combustion of the fuel.

2. Case (1) — Velocity of source is inversely proportional to the radius ($r'^2 = 2\,Vt$).

$$T - T_o = \frac{q\sqrt{2V}}{2k}\int_0^t \frac{\sqrt{t'}}{(t-t')} e^{-\frac{(r^2+2Vt')}{4\alpha(t-t')}} I_o\left\{\frac{\surd\sqrt{2Vt'}}{2\alpha(t-t')}\right\} dt' \quad . \quad . \quad . \quad . \quad (3)$$

where $V = Ur' =$ constant. . (4)

The fuel requirements may be found by integrating Equation 3 for $r = r'$ at t' and using the expression:

$$\frac{\text{lb fuel}}{\text{cubic foot}} \doteq \frac{qr'}{sV} \quad . \quad . \quad . \quad . \quad (5)$$

3. Case (2) — Wellbore effect included ($r'^2 = 2\,Vt'$)

$$T - T_o = \frac{qr'}{c\rho V}\int_1^{r'/r_w}\int_0^\infty e^{-\frac{kx^2}{2V}\left(\frac{r'^2}{r_w^2}-1\right)} \times C\left(x, \frac{r}{r_w}\right) C\left(x, \frac{r'}{r_w}\right) \times x\,dx\,d\left(\frac{r}{r_w}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where

$$C\left(x, \frac{r}{r_w}\right) =$$

$$\frac{J_0\left(\frac{xr}{r_w}\right)Y_0(x) - Y_0\left(\frac{xr}{r_w}\right)J_0(x)}{\left\{J_0^2(x) + Y_0^2(x)\right\}^{1/2}} \qquad (7)$$

In Equations 6 and 7, c is the specific heat of the saturated sand, ρ is bulk density, r_w is the radius of the wellbore, J_0 and Y_0 are Bessel functions of the first and second kind, respectively, of order zero, and x is a variable of integration.

A further refinement of the problem treated by authors Vogel and Krueger is the consideration of vertical heat losses. This would require the expensive modification of rebuilding the RC network in two dimensions, and considering a burning front of finite vertical dimension. Vertical heat loss can be important to the maintenance of self-sustained combustion when the sand section is thin and the well spacing is large. If vertical heat losses were considered, it would also be desirable to account for the lower thermal diffusivity of the "burned" sand section of the reservoir.

We agree with the authors' statement that the data developed by them in this paper should not be construed as applying directly to field operations, inasmuch as the front is known to progress over a range of temperatures, velocities, and fuel consumptions in accordance with incompletely defined rate phenomena occurring in a reservoir. Many challenging problems along these lines remain to be solved.

AUTHORS' REPLY to JENKINS and RAMEY

We should like to thank Rodman Jenkins and H. J. Ramey, Jr., for their pertinent comments. It is gratifying to see the good agreement reported between the analytical and the analog solutions to the problem of determining the fuel requirement in a radial system in which the burning front velocity is inversely proportional to the radial position. This same agreement has been noted by R. G. Hawthorne of our laboratory.

The presentation of the equations used in studying the simplified problems discussed in the paper serves to illustrate the somewhat complicated nature of analytical solutions to heat transfer during thermal oil recovery processes. Equations 3, 6, and 7 illustrate the complexity added to the simple case by considering the wellbore effects. In the computer, wellbore effects are automatically taken care of in the design of the electrical lumps near the wellbore. It would be interesting to see the analytical solutions to the more complex heat transfer problem involving the additional factors of vertical heat losses and heat transfer by the driving gases and condensing fluids.

In general, analytical solutions of this nature are tedious and time consuming unless attacked with modern digital computer techniques. In any case, considerable expense is involved in arriving at numerical answers. It would be of considerable value to the industry to have further work published on solutions to the more complex problems. On the basis of our present thinking, we feel that most such problems can be treated somewhat more simply and inexpensively by the modification of the analog computer.

Jenkins and Ramey have suggested that vertical heat losses be considered as a refinement to the problems treated in the paper. As they have noted, vertical heat losses can be important when considering the problem of maintaining a combustion front in thin oil zones. We believe, however, that the applications in which vertical heat losses are the controlling factor are likely to be the exception rather than the rule. For thicker zones there is some evidence from analytical studies that the effect of vertical heat transfer on fuel requirements is considerably less than the effect of heat transfer by injected air (or inert gases) and possibly by the flowing liquids. The effects of both of these latter factors can be studied rather easily by inexpensive modifications of the analog computer.

As Jenkins and Ramey have emphasized, radial heat flow conditions exist for only 20 to 30 per cent of the distance from the injection well toward the producing well. At the moment, we believe this region to require the most critical consideration when attempting to evaluate the feasibility of starting a successful thermal oil recovery process. However, we feel that a study of heat transfer during later stages of the process can be achieved through relatively simple modifications of the computer. It appears that analytical solutions involving interference between wells might become quite complex.

It is obvious from a consideration of both the comments of Jenkins and Ramey and the authors' reply that there is much to be studied and learned in this field. Both analytical and analog studies can be helpful in solving the many problems involved. However, as mentioned in the paper, their maximum utilization will be achieved through correlation with laboratory and field studies. ★★★